(b) Determine the equation of the hyperplane in R^9 that goes through the following nine points:

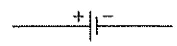
$$\begin{array}{lll} (1,2,3,4,5,6,7,8,9) & (2,3,4,5,6,7,8,9,1) & (3,4,5,6,7,8,9,1,2) \\ (4,5,6,7,8,9,1,2,3) & (5,6,7,8,9,1,2,3,4) & (6,7,8,9,1,2,3,4,5) \\ (7,8,9,1,2,3,4,5,6) & (8,9,1,2,3,4,5,6,7) & (9,1,2,3,4,5,6,7,8) \end{array}$$

11.2 ELECTRICAL NETWORKS

In this section basic laws of electrical circuits are discussed, and it is shown how these laws can be used to obtain systems of linear equations whose solutions yield the currents flowing in an electrical circuit.

PREREQUISITES: Linear Systems

The simplest electrical circuits consist of two basic components:

electrical sources	denoted by	
resistors	denoted by	

Electrical sources, such as batteries, create currents in an electrical circuit. Resistors such as lightbulbs, limit the magnitudes of the currents.

There are three basic quantities associated with electrical circuits: ***electrical potential*** (E), ***resistance*** (R), and ***current*** (I). These are commonly measured in the following units:

E	in volts	(V)
R	in ohms	(Ω)
I	in amperes	(A)

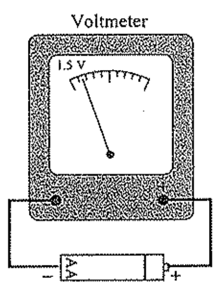


Figure 11.2.1

Electrical potential is associated with two points in an electrical circuit and is measured in practice by connecting those points to a device called a *voltmeter*. For example, common AA battery is rated at 1.5 volts, which means that this is the electrical potential across its positive and negative terminals (Figure 11.2.1).

In an electrical circuit the electrical potential between two points is called the ***voltage drop*** between these points. As we shall see, currents and voltage drops can be either positive or negative.

The flow of current in an electrical circuit is governed by three basic principles:

1. ***Ohm's Law*** The voltage drop across a resistor is the product of the current passing through it and its resistance; that is, $E = IR$.
2. ***Kirchhoff's Current Law*** The sum of the currents flowing into any point equals the sum of the currents flowing out from the point.
3. ***Kirchhoff's Voltage Law*** Around any closed loop, the algebraic sum of the voltage drops is zero.

EXAMPLE 1 Finding Currents in a Circuit

Find the unknown currents I_1, I_2, and I_3 in the circuit shown in Figure 11.2.2.

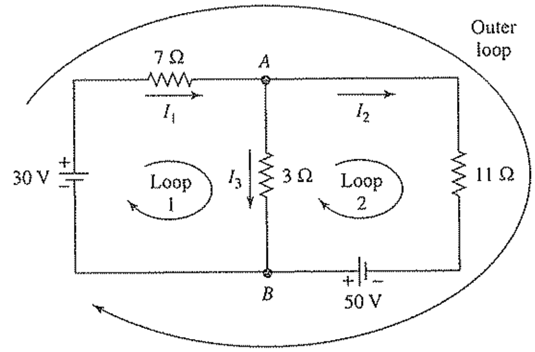


Figure 11.2.2

Solution

The flow directions for the currents I_1, I_2, and I_3 (marked by the arrowheads) were picked arbitrarily. Any of these currents that turn out to be negative actually flow opposite to the direction selected.

Applying Kirchhoff's current law to points A and B yields

$$\begin{aligned} I_1 &= I_2 + I_3 \quad &(\text{Point } A) \\ I_3 + I_2 &= I_1 \quad &(\text{Point } B) \end{aligned}$$

Since these equations both simplify to the same linear equation

$$I_1 - I_2 - I_3 = 0 \tag{1}$$

we still need two more equations to determine I_1, I_2, and I_3 uniquely. We will obtain them using Kirchhoff's voltage law.

To apply Kirchhoff's voltage law to a loop, select a positive direction around the loop (say clockwise) and make the following sign conventions:

- A current passing through a resistor produces a positive voltage drop if it flows in the positive direction of the loop and a negative voltage drop if it flows in the negative direction of the loop.
- A current passing through an electrical source produces a positive voltage drop if the positive direction of the loop is from + to − and a negative voltage drop if the positive direction of the loop is from − to +.

Applying Kirchhoff's voltage law and Ohm's law to loop 1 in Figure 11.2.2 yields

$$7I_1 + 3I_3 - 30 = 0 \tag{2}$$

and applying them to loop 2 yields

$$11I_2 - 3I_3 - 50 = 0 \tag{3}$$

Combining (1), (2), and (3) yields the linear system

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ 7I_1 \qquad + 3I_3 &= 30 \\ 11I_2 - 3I_3 &= 50 \end{aligned}$$

Solving this linear system yields the following values for the currents:

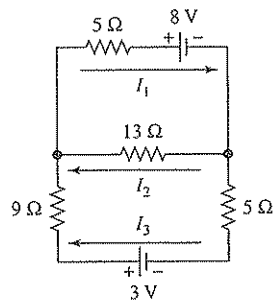
$$I_1 = \tfrac{570}{131}\ (\text{A}), \qquad I_2 = \tfrac{590}{131}\ (\text{A}), \qquad I_3 = -\tfrac{20}{131}\ (\text{A})$$

Note that I_3 is negative, which means that this current flows opposite to the direction indicated in Figure 11.2.2. Also note that we could have applied Kirchhoff's voltage law to the outer loop of the circuit. However, this produces a redundant equation (try it). ◈

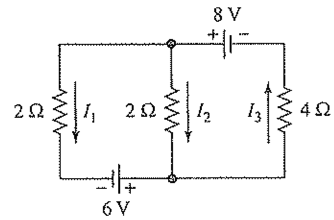
EXERCISE SET 11.2

In Exercises 1–4 find the currents in the circuits.

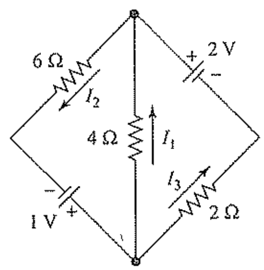
1.



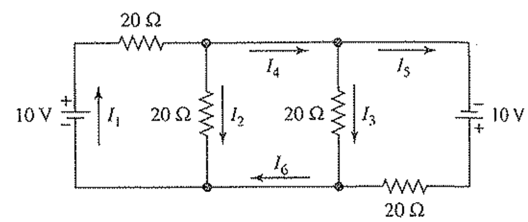
2.



3.



4.



5. Show that if the current I_5 in the circuit of the accompanying figure is zero, then $R_4 = R_3R_2/R_1$.

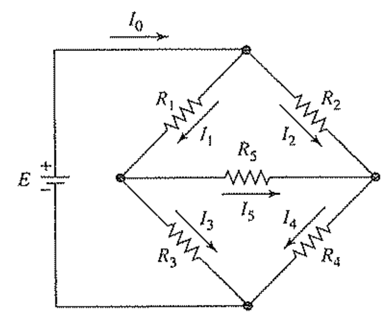


Figure Ex-5

Remark This circuit, called a Wheatstone bridge circuit, is used for the precise measurement of resistance. Here, R_4 is an unknown resistance and R_1, R_2, and R_3 are adjustable calibrated resistors. R_5 represents a galvanometer—a device for measuring current. After the operator varies the resistances R_1, R_2, and R_3 until the galvanometer reading is zero, the formula $R_4 = R_3R_2/R_1$ determines the unknown resistance R_4.

6. Show that if the two currents labeled I in the circuits of the accompanying figure are equal, then $R = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$.

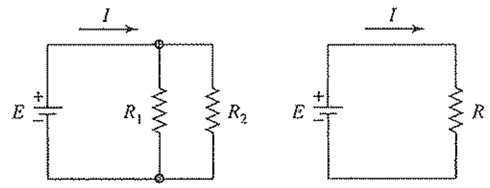


Figure Ex-6

SECTION 11.2 Technology Exercises

The following exercises are designed to be solved using a technology utility. Typically, this will be MATLAB, *Mathematica*, Maple, Derive, or Mathcad, but it may also be some other type of linear algebra software or a scientific calculator with some linear algebra capabilities. For each exercise you will need to read the relevant documentation for the particular utility you are using. The goal of these exercises is to provide you with a basic proficiency with your technology utility. Once you have mastered the techniques in these exercises, you will be able to use your technology utility to solve many of the problems in the regular exercise sets.

T1. The accompanying figure shows a sequence of different circuits.

(a) Solve for the current I_1, for the circuit in part (*a*) of the figure.

(b) Solve for the currents I_1 through I_3, for the circuit in part (*b*) of the figure.

(c) Solve for the currents I_1 through I_5, for the circuit in part (*c*) of the figure.

(d) Continue this process until you discover a pattern in the values of $I_1, I_2, I_3, \ldots$.

(e) Investigate the sequence of values for I_1 in each of the circuits in parts (a), (b), (c), and so on, and numerically show that the limit of this sequence approaches the value

$$\left(\frac{\sqrt{5}-1}{2}\right)\frac{E}{R} \approx (0.6180)\frac{E}{R}$$

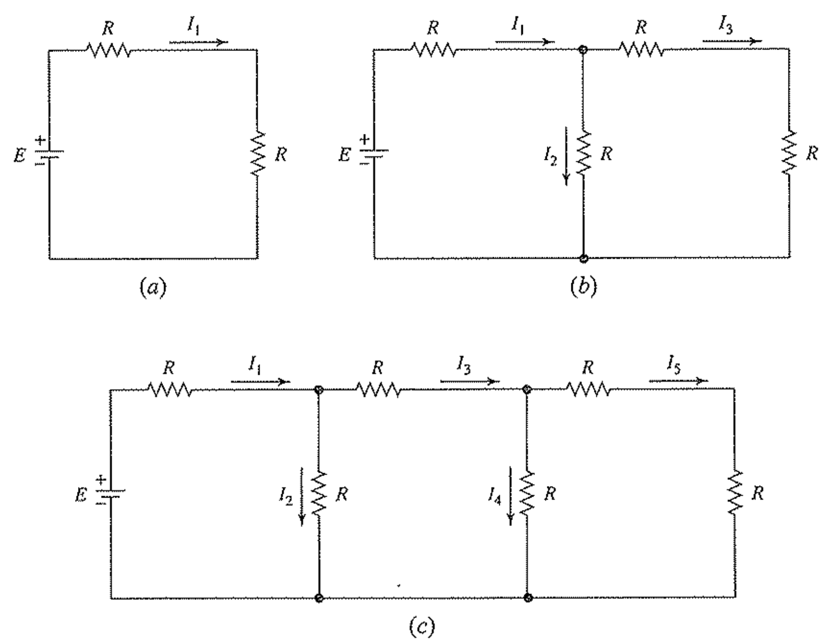


Figure Ex-T1

T2. The accompanying figure shows a sequence of different circuits.

(a) Solve for the current I_1, for the circuit in part (a) of the figure.

(b) Solve for the current I_1, for the circuit in part (b) of the figure.

(c) Solve for the current I_1, for the circuit in part (c) of the figure.

(d) Continue this process until you discover a pattern in the values of I_1.

(e) Investigate the sequence of values for I_1 in each of the circuits in parts (a), (b), (c), and so on, and numerically show that the limit of this sequence approaches the value

$$\left(\frac{\sqrt{5}-1}{2}\right)\frac{E}{R} \approx (0.6180)\frac{E}{R}$$

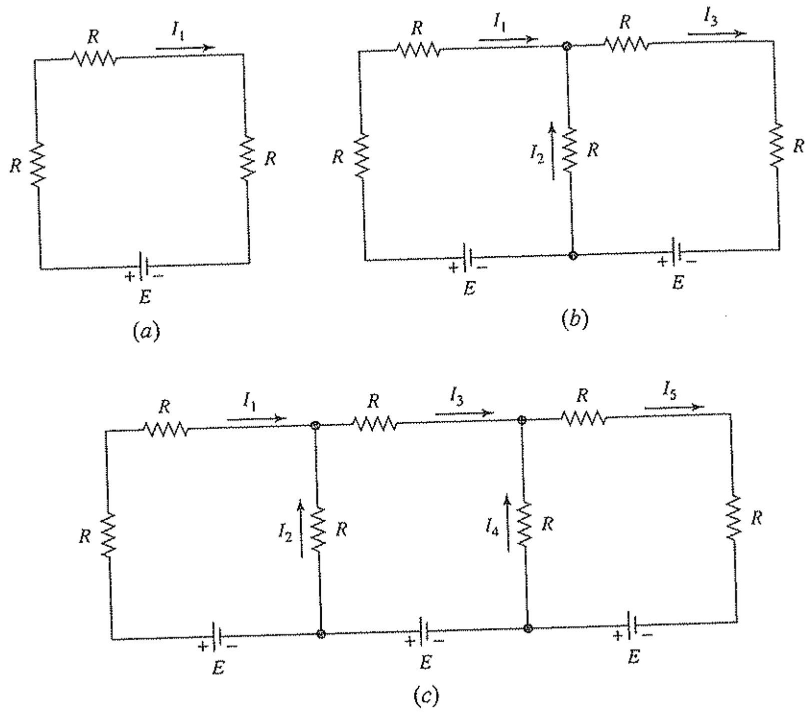


Figure Ex-T2

11.3 GEOMETRIC LINEAR PROGRAMMING

In this section we describe a geometric technique for maximizing or minimizing a linear expression in two variables subject to a set of linear constraints.

PREREQUISITES: Linear Systems
Linear Inequalities

Linear Programming

The study of linear programming theory has expanded greatly since the pioneering work of George Dantzig in the late 1940s. Today, linear programming is applied to a wide variety of problems in industry and science. In this section we present a geometric approach to the solution of simple linear programming problems. Let us begin with some examples.