Review Problem 2.1 A random variable X has the pdf given below. For each, (a) verify that $f(x)$ is a pdf, (b) find $E(X)$, and (c) find $\text{Var}(X)$.

1. $f(x) = 3(1-x)^2, \qquad 0 \le x \le 1$
2. $f(x) = \begin{cases} 3/4, & 0 \le x \le 1, \\ 1/4, & 2 \le x \le 3, \\ 0, & \text{otherwise.} \end{cases}$

Review Problem 2.2 Suppose 15 independent observations are chosen at random from the pdf $f(x) = 3x^2$, where $0 \le x \le 1$. Let N denote the number of these that lie in the interval $(1/2, 1)$. Find $E(N)$.

Review Problem 2.3 A die is rolled until a 6 appears. What is the probability that more than four rolls will be required?

Review Problem 2.4 Let $X \sim \text{Geometric}(p)$. Compute $P(X > n)$. Then show that

$$P(X = n + k | X > n) = P(X = k)$$

if $k \ge 1$.

Review Problem 2.5 Assume that X and Y are independent random variables. Prove that

$$\text{Var}(X + Y) = \text{Var}(X - Y)$$

Review Problem 2.6 Let D_1 and D_2 be i.i.d. random variables with $E(D_1) = 10$ and $\text{SD}(D_1) = 2$. Find a number c so that

$$P(|D_1 - D_2| < c) \ge 0.99$$

Review Problem 2.7 A box contains w white balls and b black balls. Balls are drawn one by one at random from the box until b black balls have been drawn. Let X be the number of draws made. Find the distribution of X if the draws are made with replacement.

Review Problem 2.8 Repeat the previous problem if the draws are made without replacement.

Review Problem 2.9 A fair die is rolled. Let X be the highest power of 2 that divides the number that appears.

1. Find the probability frequency function for X.
2. Find $E(X)$ and $\text{SD}(X)$.
3. Find $E(-5X)$ and $\text{SD}(-5X)$.

Review Problem 2.10 Phone calls arrive into a phone bank according to a Poisson process with rate 2 per minute.

1. Find the probability that at least three calls arrive in the first minute.
2. Let X be the time between the third and fourth calls. Find the expected value and standard deviation of X.
3. Let X be the number of calls in the first two minutes, and let Y be the number of calls in the first four minutes. Compute $P(X = k \mid Y = 10)$. Recognize this as a common distribution, and state its parameters.

Review Problem 2.11 Suppose a gambler plays roulette, betting on his favorite number. If his number comes up (with probability 1/38), then he wins \$35. However, if his number doesn't come up, he loses \$1.

1. Let X be the winnings on one play. Find $E(X)$ and $\mathrm{SD}(X)$.
2. Suppose he plays 2500 times; let Y denote his total winnings. Find $E(Y)$ and $\mathrm{SD}(Y)$.
3. If he plays 2500 times, find the probability that he has a positive net gain.

Review Problem 2.12 Bullets are fired at a target. Suppose the distribution of the impact marks are Poisson distributed with rate 2 per square foot. Find the expected distance of the closest mark to the bullseye. (Assume that the impact marks and the bullseye are all points instead of circles.) *Hint:* $E(D) = \int_0^\infty P(D \geq r)\,dr$.

Review Problem 2.13 Suppose a particular kind of atom has a half-life of 10 years. Find

1. The probability an atom survives at least 30 years
2. The time required for 90% of all such atoms to decay
3. The lifetime's mean and standard deviation

Review Problem 2.14 Find the median of the random variable with pdf

$$f(x) = 2x, \qquad 0 \leq x \leq 1$$

Review Problem 2.15 Let N have mean μ and standard deviation σ.

1. Simplify $E[(N-x)^2]$. Your answer should involve μ, σ, and x.
2. Find the value of x that minimizes $E[(N-x)^2]$.
3. Find the smallest value of $E[(N-x)^2]$.

Review Problem 2.16 Let X have pdf

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2. \end{cases}$$

1. Find $P(X \leq 1.5)$.
2. Find the 90th percentile of the distribution.

Review Problem 2.17 Let X have a normal distribution with mean 25 and standard deviation 6. Find the number c so that $P(|X - 25| \leq c) = 0.8$.

Review Problem 2.18 Let X be a random variable with mean μ and standard deviation σ. Find $P(\mu - 2\sigma < X < \mu + 2\sigma)$ if

1. $X \sim$ Poisson(5)
2. $X \sim$ Exponential(5)
3. $X \sim$ Normal(16,3)
4. Use Chebyshev's inequality to find the smallest possible value of

$$P(\mu - 2\sigma < X < \mu + 2\sigma).$$

Review Problem 2.19 Grandma bakes chocolate chip cookies in batches of 100. She puts 300 chips into the dough. When the cookies are done, she gives you one. What is the probability that your cookie contains no chips?

Review Problem 2.20 Consider a Poisson process with arrival rate λ per second. Let X be the number of arrivals in t seconds, and let $L = X/t$. Find $E(L)$ and $\mathrm{SD}(L)$.

Review Problem 2.21 In the game chuck-a-luck, three fair dice are rolled after you pick a number. You win a dollar for every time your number appears. But if your number does not appear, you lose a dolalr.

1. Let X be the amount you win after playing the game once. Find $E(X)$ and $\mathrm{SD}(X)$.
2. Suppose you play the game 100 times. Let Y be your total winnings. Find $E(Y)$ and $\mathrm{SD}(Y)$.

Review Problem 2.22 Let X be the number of times that a fair coin, flipped 50 times, lands heads. Use the normal approximation to estimate $P(X = 25)$. Then compare your answer with the exact answer.

Review Problem 2.23 A light fixture contains five light bulbs. The lifetime of each bulb is exponentially distributed with mean 200 hours. Assuming that the lifetimes of the bulbs are independent, find the probability that all five light bulbs last at least 100 hours.

Review Problem 2.24 Let $X \sim$ Binomial(n,p). Let $P = X/n$. Find $E(P)$ and $\mathrm{SD}(P)$.

Review Problem 2.25 A school class of 120 students are taken on a field trip in 3 busses. There are 36 students on one bus, 40 on the second, and 44 on the third. After the field trip, one of the 120 students is chosen at random. Let X be the number of students on the bus of the chosen student. Find $E(X)$.