

## Math 4050

## Practice Problem Set #14

*At the top of your write-up, you must also write a statement attesting that you have at least thought about all assigned problems. Points will be deducted if you do not write this statement. This does not mean that you solved all of the problems — just that you gave some thought about how to solve every problem. For the sake of preparing for the state certification exam, as well as for your own integrity, I'd prefer that you are honest when writing this statement.*

**Problem 14.1** A college professor has 25 dress shirts and 30 ties. How many outfits are possible?

**Problem 14.2** A race has thirteen contestants. The winner receives a gold medal, second-place receives a silver medal, and third-place receives a bronze medal. How many different ways are there of awarding these medals if all possible outcomes can occur and there are no ties?

**Problem 14.3** A salesman has to visit eight different cities. The first city is specified, but he can visit the other seven cities in any order. How many trips are possible?

**Problem 14.4** How many poker hands of 5 cards can be dealt from a standard deck of 52 cards?

**Problem 14.5** A mathematics department has 24 faculty members, and a physics department has 20 faculty members. The dean wishes to form a committee of three members from each department (for a total of six). How many committees are possible?

**Problem 14.6** How many “runs” (like HHHHHH) of flipping a coin six times are possible?

**Problem 14.7** How many “words” of length  $r$  can be formed from the English alphabet (whether or not they appear in the dictionary)?

**Problem 14.8** Prove that  $\binom{n}{r} = \binom{n}{n-r}$ .

**Problem 14.9** Prove that  $\sum_{k=0}^n \binom{n}{k} = 2^n$ .

**Problem 14.10** Prove that  $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$ .

**Problem 14.11** How many strings can be made from the letters in TEXAS (using all letters)?

**Problem 14.12** How many strings can be made from the letters in TEACHNORTHTEXAS (using all letters)?

**Problem 14.13** How many ways are there to distribute 5 cards each to six poker players from a 52-card deck?

**Problem 14.14** In a certain class, all students have blond hair, blue eyes, or both. The number of blonds is 25, the number with blue eyes is 13, and the number with both is 8. How many students are in the class?

**Problem 14.15** How many positive integers not exceeding 1000 are divisible by 7 or 11 (or both)?

**Problem 14.16** A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages?