Trigonometric identities and the Weyl character formula

You may have seen trigonometric identities relating ratios of sine functions to sums of cosine functions, for example,

$$\frac{\sin 6\theta}{\sin \theta} = 2\cos 5\theta + 2\cos 3\theta + 2\cos \theta.$$

Such identities are special cases of the *Weyl character formula,* which describes representations of compact Lie groups and relates ratios of certain *generalized* sine functions to sums of generalized cosine functions. These generalized trigonometric functions depend on more than one angle, and have symmetry properties under permutation of their arguments analogous to the symmetries of sine and cosine under $\theta \mapsto -\theta$.

In this talk we will give a gentle introduction to groups and their representations. We will exhibit the Weyl character formula for the group U_3 (the 3 by 3 unitary matrices), and for the unit sphere S^3 in 4-dimensional space, which is a group under Hamiltonian (quaternionic) multiplication. In the case of S^3, we will use integration in 4-dimensional spherical coordinates to describe the approach Weyl used to arrive at his formula.