These practice problems are in roughly the same format as the exam. On the exam itself, just as on Exam I, show all work, and label all intercepts and corners of pictures and graphs. You may do the problems in any order, so begin with those you find easiest.

1. Compute

$$\int_{-\infty}^{\infty} \frac{\sin x \, dx}{x^2 - 2x + 2}$$

2. Compute

$$\int_{-\infty}^{\infty} \frac{x^2 \, dx}{(x^2 + 1)^2}$$

3. Compute

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta} \qquad \text{[Hint: expand } (3z - i)(z - 3i).]$$

4. Let $f(z) = z^{-2}(e^{2z} - 1)$.

(a) Find the first four non-zero terms of the Laurent series of f at $z = 0$.

(b) Find $\mathrm{Res}_0\, f$.

(c) Compute $\oint_{C_1(0)} f(z)\, dz$.

5. Let $g(z) = z(e^{2z} - 1)$.

(a) Find all of the zeroes of g.

(b) Find the order of each of the zeroes.

6. Let $h(z) = z^{-6}(\sin z - z\cos z)$.

(a) Find the first *three* non-zero terms of the Laurent series of h at $z = 0$.

(b) On what region does the series converge?

(c) Compute $\oint_{C_3(i)} h(z)\, dz$.

7. Find the first four non-zero terms of the Taylor series at $z = 0$, and give the region on which it converges:

(a) $\dfrac{1}{1-2z^3}$ (b) $(1+3z)^{5/3}$

8. Suppose that $k(z)$ is analytic on $D_3(2)$ and $|k(z)| \leq 5$ on $C_2(2)$.

(a) Find a bound for $|k''(2)|$.

(b) Find a bound for $|k(z)|$ on $C_1(1)$.

(b) Find a bound for $|k'(1)|$.

9. Compute $\oint_{C_\pi(0)} \tan z \, dz$.

10. Compute the following integrals:

(a) $\displaystyle\oint_{C_1(0)} z^3 \cos(2z^{-2}) \, dz$ (b) $\displaystyle\oint_{C_\pi(0)} \frac{dz}{z(1-\cos z)}$

11. Consider the following two paths running along the circle $C_{\sqrt{2}}(1)$ from $-i$ to i:

- S_+ runs counterclockwise three quarters of the way around,
- S_- runs clockwise one quarter of the way around.

Compute:

(a) $\displaystyle\int_{S_+} \frac{dz}{|z-1|^2}$ (b) $\displaystyle\int_{S_+} \frac{dz}{z^2-1}$ (c) $\displaystyle\int_{S_-} \frac{dz}{z^2-1}$

Extra Credit. Compute:

(a) $\operatorname{Res}_0 \exp(z + z^{-1})$ (b) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx$ (c) $\operatorname{Res}_0 (z^{n+1} - z^{n+2} - z^{n+3})^{-1}$