Math 4520 **Exam II Review Sheet** Conley

These practice problems are in roughly the same format as the exam. On the exam itself, just as on Exam I, show all work, and label all intercepts and corners of pictures and graphs. You may do the problems in any order, so begin with those you find easiest.

Problem 1.

(a) Compute $\sum_{n=1}^{\infty}\left(-\frac{i}{3}\right)^n$

(b) Find the radius of convergence of $f(z) = \sum_{n=1}^{\infty} \frac{z^n}{2^n \sqrt{n}}$

(c) Continuing (b), compute the power series for f' in the form $\sum_{n=0}^{\infty} b_n z^n$

Problem 2.

(a) Solve $e^z = -\sqrt{3} + i$

(b) Draw $\exp\left\{x < 0, -\frac{\pi}{6} < y < \frac{\pi}{3}\right\}$

Problem 3.

(a) Compute $\text{Log}(1 - i)$

(b) Draw $\text{Log}\left\{re^{i\theta} : r > e, \frac{\pi}{2} < \theta < \frac{3\pi}{2}\right\}$

Problem 4.

(a) Compute $\sin\left(\frac{\pi}{4} - \frac{i}{2}\ln 2\right)$

(b) Solve $\cos z = i$

Problem 5. Let A be the curve that runs straight from 0 to $\pi/2$, and then counterclockwise along $C_{\pi/2}(0)$ from $\pi/2$ to $i\pi/2$.

(a) Compute $\int_A \bar{z}\, dz$

(b) Compute $\int_A e^z\, dz$

Problem 6. Compute the following integrals:

(a) $\oint_{C_1(-1)} e^{\sin z}\, dz$ (b) $\oint_{C_1(-1)} \dfrac{dz}{1+2z}$ (c) $\oint_{C_1(-1)} \dfrac{dz}{(1+2z)^2}$

Problem 7. Compute the following integrals:

(a) $\oint_{C_2(1)} \dfrac{2\, dz}{1+z^2}$ (b) $\oint_{C_2(2i)} \dfrac{2\, dz}{1+z^2}$

Extra Credit.

(a) Compute $\operatorname{Re}\operatorname{Pr}(1-i)^i$

(b) Compute $\oint_{C_2(2i)} \dfrac{2\, z dz}{1+z^2}$

(c) Compute $\oint_{C_2(0)} \operatorname{Log} z\, dz$