4450 formulas

$\mathbb{R}^n$: vertical n-vectors. Size $m\times n$ matrices: m high, n wide.

Notation: A $m\times n$, B $l\times m$, $x\in\mathbb{R}^n$, $y\in\mathbb{R}^m$.

Transpose: flip over main diag, so A^T $n\times m$, $(A^T)_{ji}=A_{ij}$.

Matrix $\cdot$ Vector multiplication: $A:\mathbb{R}^n\to\mathbb{R}^m$, $Ax = x_1 Col_1 A+\cdots+x_n Col_n A$.

i^{th} entry of Ax: $(Ax)_i=(Row_i A)x$ ($1\times n$ mat times n-vector).

Column Space of A: $Col(A)=Span\{Col_1 A,\cdots,Col_n A\}$

Matrix Multiplication: $BA=B\circ A:\mathbb{R}^n\to\mathbb{R}^l$ is $l\times n$.

$Col_j(BA)=B\,Col_j A$, $Row_i(BA)=(Row_i B)A$, $(BA)_{ij}=(Row_i B)(Col_j A)$,

$BA=(Col_1 B)(Row_1 A)+\cdots+(Col_m B)(Row_m A)$

Row vectors: transposes of column vectors. $y^TA=y_1 Row_1 A+\cdots+y_m Row_m A$.

Standard vectors: $e_j=\begin{pmatrix}0\\ \vdots\\ 0\\ 1\\ 0\\ \vdots\\ 0\end{pmatrix}$, column vec, 1 in j^{th} spot, else 0's, height given by context.

$Ae_j=Col_j A$, $e_i^TA=Row_i A$, $e_ie_j^T=m\times n$ matrix, 1 in (ij) spot, else 0's.

e_ix^T: $m\times n$ matrix, $Row_i=x^T$, other rows 0. ye_j^T: $m\times n$, $Col_j=y$, else 0.

$e_ie_j^TA=e_i\,Row_jA=$ matrix with i^{th} row $=Row_j A$, other rows 0.

$Ae_ie_j^T=(Col_iA)e_j^T=$ matrix with j^{th} col $=Col_iA$, other cols 0.

Transpose of product: $(AB)^T=B^TA^T$. Note: $Row_j(A^T)=(Col_jA)^T$, $Col_i(A^T)=(Row_iA)^T$

Inverse of product: $(AB)^{-1}=B^{-1}A^{-1}$ if A,B both $n\times n$ invertible.

Facts: $(A^T)^T=A$. If A invertible, then $(A^{-1})^{-1}=A$, $(A^{-1})^T=(A^T)^{-1}$.

Products of elementary matrices: if e_j, e_k are same size, then

$(e_ie_j^T)(e_ke_l^T)=e_i(e_j^Te_k)e_l^T=e_ie_l^T$ if $j=k$ and 0 if $j\neq k$.