Math 3410 **Exam 2 Review Sheet** Conley

The exam will have about 7 problems and one extra-credit problem. You will need to simplify all answers as much as possible. You will be given the following formulas:

$$\mathcal{L}\big[f\big](s) = \int_0^\infty e^{-st} f(t)dt, \qquad \mathcal{L}\big[e^{-at}f\big](s) = \mathcal{L}\big[f\big](s+a),$$

$$\mathcal{L}\big[tf\big] = -\tfrac{d}{ds}\mathcal{L}\big[f\big], \qquad \mathcal{L}\big[u_a(t)f(t-a)\big](s) = e^{-as}\mathcal{L}\big[f\big](s),$$

$$\mathcal{L}\big[y'' + by' + cy\big](s) = (s^2 + bs + c)Y(s) - (s+b)y(0) - y'(0).$$

1. Consider $y'' + 3y' - 10y = g(t)$ with ICs $y(0) = 1$, $y'(0) = 2$.

(a) Let $g = 84e^{-t}$. Find $Y(s)$, apply partial fractions, and then find $y(t)$.

(b) Let $g = 49e^{2t}$. Find $Y(s)$, apply partial fractions, and then find $y(t)$.

2.

(a) Find $\mathcal{L}\big[e^{-3t}(\sin 2t + \cos 2t)\big]$.

(b) Find $\mathcal{L}\big[t^2 e^{-2t}\big]$.

(c) Find $\mathcal{L}\big[h\big]$, where $h(t) = 1$ for $1 \le t < 2$, $h(t) = 2$ for $2 \le t < 3$, and $h(t) = 0$ otherwise.

3.

(a) If $Y(s) = \frac{2s-3}{s^2+2s+10}$, what is $y(t)$?

(b) If $Y(s) = \frac{e^{-2s}}{s-2}$, what is $y(t)$?

4. Consider the equation $(1 + x^2)y'' - 2y = 0$.

(a) Find the recursion relation for the power series at $x = 0$ of its solutions.

(b) Find the first three non-zero terms of each of its fundamental solutions.

5. Find the general solution of $2x^2y'' + 3xy' - y = 0$.

6. Suppose that $2x^2y'' + (3x - x^2)y' - y = 0$ has solution

$$y_r = x^r + a_{r+1}x^{r+1} + a_{r+2}x^{r+2} + \cdots .$$

(a) What are the possible values of r?

(b) What is the recursion relation for the a_n? (It does not depend on r.)

(c) For each possible value of r, compute a_{r+1} and a_{r+2}.

7. Suppose that $y'' - y\sqrt{1-2x} = 0$ has ICs $y(0) = y'(0) = 1$ and solution

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots .$$

Find a_0, a_1, a_2, and a_3.

Extra Credit.

(a) Find the general solution in Problem 6 in terms of an indefinite integral.

(b) What can you say about the general solution of $x^3y'' - y = 0$?