Answer Key

Math 1710.200- Exam 1

Put your name on the top right of the page.

1. (15 points) Evaluate the following limit or state that it does not exist

$$\lim_{x\to 1} \frac{x-1}{x-\sqrt{x}}$$

$$\lim_{x\to 1} \frac{x-1}{x-\sqrt{x}} = \lim_{x\to 1} \frac{x-1}{x-\sqrt{x}}\left(\frac{x+\sqrt{x}}{x+\sqrt{x}}\right)$$

$$= \lim_{x\to 1} \frac{(x-1)(x+\sqrt{x})}{x^2-x} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+\sqrt{x})}{x\cancel{(x-1)}}$$

$$= \lim_{x\to 1} \frac{x+\sqrt{x}}{x} = \frac{1+\sqrt{1}}{1} = \frac{2}{1} = 2$$

2. (15 points) Find the vertical, horizontal, and slant asymptotes of the following function, if they exist. Sketch a graph of the function.

$$f(x) = \frac{3x^2-27}{x^2+5x} = \frac{3(x-3)(x+3)}{x(x+5)}$$

$\lim_{x\to 0^+} f(x) = -\infty$

$\lim_{x\to 0^-} f(x) = \infty$

$\lim_{x\to -5^+} f(x) = -\infty$

$\lim_{x\to -5^-} f(x) = \infty$

V.A.s when $x^2+5x=0$

$x^2+5x = x(x+5) = 0$

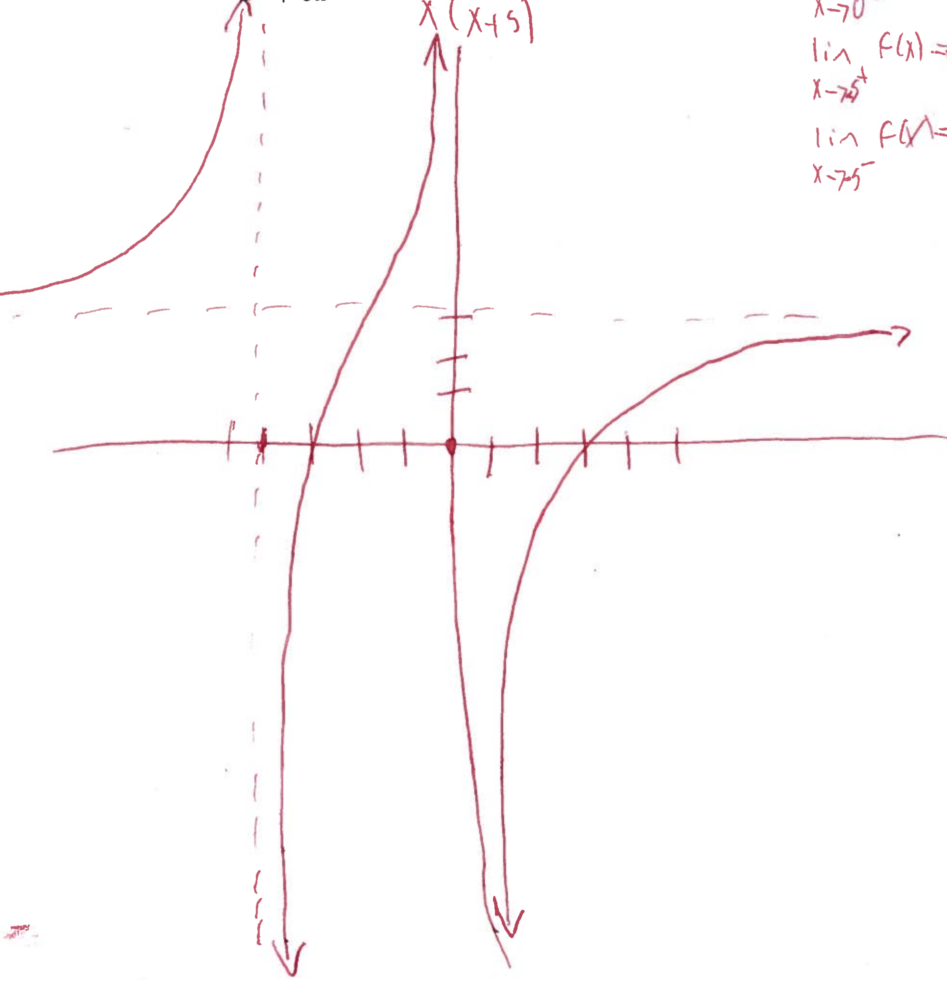
V.A.s at $x=0$, $x=-5$

H.A.s

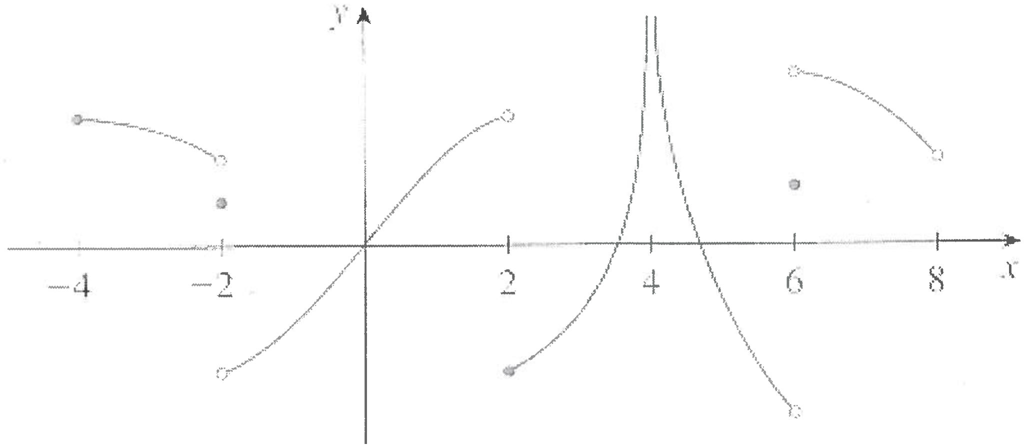
Power of x in numerator is same as power of x in denominator. So H.A. at $y=\frac{3}{1}=3$

S.A.

No slant asymptotes



3. (10 points) On what intervals in the function f in the picture continuous?



$[-4,2), (-2,2), [2,4), (4,6), (6,8)$

4. (30 points) Evaluate the following derivatives

(a) $\frac{d}{dx}(x^2 \sin x + \frac{\sqrt{x}+1}{cos x})$

$$x^2 \cos x + 2x \sin x + \frac{\cos x(\frac{1}{2}x^{-1/2}) - (\sqrt{x}+1)(-\sin x)}{\cos^2 x}$$

(b) $\frac{d}{dx}(\sin^2 x + \sqrt{x^2+1})$

$$2\sin x \cdot \cos x + \frac{1}{2}(x^2+1)^{-1/2}(2x) = 2\sin x \cos x + (x^2+1)^{-1/2}$$

7. BONUS (5 points) Recall that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Use this fact to evaluate the following limit

$$\lim_{x\to 0} \frac{\sin^3 x}{\sin x^3}$$

$$\lim_{x\to 0} \frac{\sin^3 x}{\sin x^3} = \lim_{x\to 0} \frac{\frac{\sin^3 x}{x^3}}{\frac{\sin x^3}{x^3}}$$

(1) $$\lim_{x\to 0} \frac{\sin^3 x}{x^3} = \lim_{x\to 0} \frac{\sin x \cdot \sin x \cdot \sin x}{x \cdot x \cdot x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{\sin x}{x} = 1 \cdot 1 \cdot 1 = 1$$

(2) $$\lim_{x\to 0} \frac{\sin x^3}{x^3} \underset{t=x^3}{=} \lim_{t\to 0} \frac{\sin t}{t} = 1$$

so $$\lim_{x\to 0} \frac{\sin^3 x}{\sin x^3} = \lim_{x\to 0} \frac{\left(\frac{\sin^3 x}{x^3}\right)}{\left(\frac{\sin x^3}{x^3}\right)} = \frac{\lim_{x\to 0}\left(\frac{\sin^3 x}{x^3}\right)}{\lim_{x\to 0}\left(\frac{\sin x^3}{x^3}\right)} = \frac{1}{1} = 1$$

5. (15 points) Find $\frac{dy}{dx}$ and write an equation for the tangent line at $(1, 8)$.

$$x\sqrt[3]{y} + y = 10$$

Find $\frac{dx}{dx}$:

$\frac{d}{dx}(x\sqrt[3]{y}) + \frac{d}{dx}(y) = \frac{d}{dx}(10)$

$x\frac{1}{3}y^{-2/3}y' + \sqrt[3]{y} + y' = 0$

$y'(x\frac{1}{3}y^{-2/3} + 1) = -\sqrt[3]{y}$

$\frac{dy}{dx} = y' = \frac{-\sqrt[3]{y}}{\frac{1}{3}xy^{-2/3}+1}$

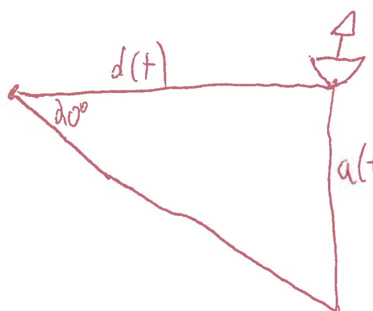
Slope of tangent line at $(1,8)$ is

$\frac{-\sqrt[3]{8}}{\frac{1}{3}(1)(8^{-2/3})+1} = \frac{-2}{\frac{1}{3}(1)\frac{1}{4}+1} = \frac{-2}{\left(\frac{13}{12}\right)} = -\frac{24}{13}$

Line with slope $\frac{-24}{13}$ through $(1,8)$

$y - 8 = \frac{-24}{13}(x-1)$

6. (15 points) A ship is moving on the surface of the ocean in a straight line at $10km/hr$. At the same time, a submarine maintains a position directly below the ship while diving at an angle of 20 degrees below the horizontal. How fast is the submarine's altitude decreasing?



$d(t)$ = distance of ship = $10t$ after t hours

$a(t)$ = distance of submarine below water.

The question is asking "What is $a'(t)$?"

$\tan(20°) = \frac{a(t)}{d(t)}$

$\tan(20°) = \frac{a(t)}{10t}$

$a(t) = \tan(20°)10t$

$a'(t) = \tan(20°)\cdot 10$

The submarine's altitude is decreasing at a rate of $\tan(20°)\cdot 10$ km/hr