Derivatives Question 1 (200 points)

Use the definition of derivative to show that if $f(x) = x^2$, then $f'(x) = 2x$

Derivatives Question 2 (200 points)

Let

$$f(x) = x^2 \sin(\pi\sqrt{x})$$

Find the equation to the tangent line to f at $x = 1$.

Derivatives Question 3 (350 points)
Let

$$g(x) = \tan^2(\sqrt{x^2 + \sqrt{x}})$$

What is $g'(x)$?

Derivatives Question 4 (400 points)

Find $\frac{dy}{dx}$ for the following curve

$$\cos(x - y) + \sin y = \sqrt{2}$$

Derivatives Question 5 (400 points)
Find $\frac{dy}{dx}$ for the following curve

$$(x^2 + y^2)^2 = \frac{25}{4}xy^2$$

Derivatives Question 6 (500 points)

Sand falls from an overhead bin and accumulates in a conical poile with a radius that is always 3 times its height. Suppose the height of the pile increases at a rate of $2\frac{cm}{s}$ when the pile is 12 cm high. At what rate is the sand leaving the bin at that instant?

The volume of a cone of height h and a circular base of radius r is $V = \frac{1}{3}\pi r^2 h$.

Derivatives Question 7 (400 points)

A hot air balloon is $150ft$ above the ground when a motorcycle (traveling in a straight line on a horizontal road) passes directly beneath it going $40mph$ $(58.67\frac{ft}{s})$. If the balloon rises vertically at a rate of $10\frac{ft}{s}$, what is the rate of change of the distance between the motorcycle and the balloon ten seconds later?

Applications of Derivative Questions 1 (400 points)

A square-based, box-shaped shipping crate is designed to have a volume of $16ft^3$. The material used to make the base costs twice as much (per square foot) as the material in the sides. The material for the top costs half as much as the material for the sides (per square foot). What are the dimensions that minimize the cost of the crate?

Applications of Derivatives Question 2 (400 points)

A rectangular flower garden with an area of $30m^2$ is surrounded by a grass border that is $1m$ wide on two sides and $2m$ wide on the other two sides. What dimensions of the garden minimize the combined area of garden and borders?
(Draw picture)

Applications of Derivatives Question 3 (450 points)

Consider $f(x) = 2x - 3x^{2/3}$. Sketch a graph by hand by completing the following steps.

1. Determine the end behavior (as $x \to \pm\infty$).
2. Determine where f is increasing and decreasing, and identify any local extrema.
3. Determine where f is concave up or down and identify any inflection points.
4. Sketch the graph and label all the relevant points.

Applications of Derivatives Question 4 (350 points)

Sketch a graph of possible graph of f given the following graphs of f' and f'' (draw picture)

Applications of Derivatives Question 5 (350 points)
Evaluate the following limit

$$\lim_{x \to 0} \frac{\sec x - 1}{x^2}$$

Applications of Derivatives Question 6 (350 points)
Evaluate the following limit

$$\lim_{x\to\infty} x^2 \sin(\frac{1}{4x^2})$$

Integrals Question 1 (300 points)
Recall the FTOC.
Evaluate the following expression:

$$\frac{d}{dx}\int_{2}^{x^2}\frac{dp}{p^2}dp$$

Integrals Question 2 (300 points)
Evaluate the following definite integral:

$$\int_0^{\frac{6}{5}} \frac{x}{(25x^2+36)^2} dx$$

Integrals Question 3 (350 points)
Evaluate the following indefinite integral:

$$\int \frac{\sec^2 x}{\tan^3 x} dx$$

Integrals Question 4 (350 points)
Evaluate the following indefinite integral:

$$\int (z+1)\sqrt{3z+2}dz$$

Applications of Integration Question 1 (300 points)
Find the area bounded by

$$y = \sin x$$

and

$$y = \sin 2x$$

between $x = 0$ and $x = \pi$.

Applications of Integration Question 2 (300 points)
Find the area bounded by $x = 2y$ and $x = y^2 - 3$

Applications of Integration Question 3 (450 points)
Find the area of the solid whose base is the region bounded by the curve

$$y = \sqrt{\cos x}$$

and the x-axis on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and whose cross-sections through the solid perpendicular to the x-axis are isosceles right triangles.
(draw picture)

Applications of Integration Question 4 (400 points)
Let R be the region bounded by $y = 0$, $y = \sqrt{x-1}$ $y = 2$, and $x = 0$. Find the volume of the solid that comes from rotating R about the y-axis.

Applications of Integration Question 5 (450 points)

Let R be the region bounded by $y = \frac{4}{\sqrt{x+1}}$, $y = 1$, and $x = 0$. Compute the volume of the sold that comes from rotating R about the y-axis

Applications of Integration Question 6 (350 points)
Let R be the region bounded by $y = \sec x$, $y = 2$ and $x = 0$. Compute the volume of the solid that comes from rotating R about the x-axis.

Applications of Integration Question 7 (400 points)

Let R be the region bounded by $y = (1-x)^{-\frac{1}{2}}$, $y = 1$, and $y = 2$. Compute the volume of the solid that comes from rotating R about the y-axis.

Applications of Integration Question 8 (350 points)
Let R be the region bounded by $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid that comes from rotating R about the x-axis.

Applications of Integration Question 9 (500 points)

Let R be the region bounded by $x = \frac{1}{2\sqrt{y}}$, $x = \frac{1}{2}$ and $y = \frac{1}{4}$. Find the volume of the solid that comes from rotating R about the x-axis.