

Name: _____

Solution

MATH 1710 - 4.1 and 4.2 worksheet

Write legibly and show all your work. You may work together, use your book/notes, and receive guidance from your TA. However, you must write your own solutions.

- Find the absolute maximum and minimum of $f(x) = -x^2 + x - 4$ on $[-4, 4]$.

$$\begin{aligned} \text{C.p.} \quad f'(x) &= -2x + 1 \\ 0 &= -2x + 1 \\ x &= \frac{1}{2} \end{aligned}$$

$$f(-4) = -24 \leftarrow \text{min } (-4, -24)$$

$$f(4) = -16$$

$$f\left(\frac{1}{2}\right) = -3\frac{3}{4} \leftarrow \text{max } \left(\frac{1}{2}, -3\frac{3}{4}\right)$$

- Consider $f(x) = \frac{8x}{x^2 + 4}$. Sketch a graph by hand by completing the following steps.

- What are the zeros of f ?

$$\text{Zero at } x = 0$$

- Determine the end behavior (as $x \rightarrow \pm\infty$).

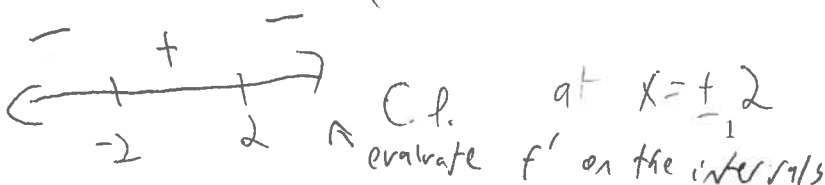
$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

- Find the critical points of f . Determine where f is increasing and decreasing, and identify any local extrema.

$$f'(x) = \frac{8(x^2 + 4) - 8x(2x)}{(x^2 + 4)^2} = \frac{8x^2 + 32 - 16x^2}{(x^2 + 4)^2} = \frac{-8x^2 + 32}{(x^2 + 4)^2} = \frac{-8(x^2 - 4)}{(x^2 + 4)^2}$$

decreasing increasing decreasing



local min at $x = -2$
local max at $x = 2$

(d) Find the inflection points of f . Determine where f is concave up or down.

evaluate f'' on the intervals

$$f'(x) = \frac{-8(x^2-4)}{(x^2+4)^2}$$

Inflection points at
 $x = 0, \pm 2$

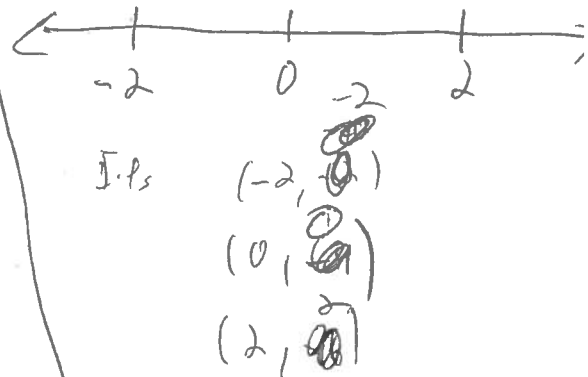
concave down concave up concave down concave up
- + - +

$$f''(x) = \frac{(x^2-4)^2(-16x) - (-8x^2+32)2(x^2+4)2x}{(x^2+4)^4}$$

$$= \frac{(x^4-8x^2+16)(-16x) - (-8x^2+32)(4x^3+8x)}{(x^2+4)^4}$$

$$= \frac{(-16x^5+128x^3-256x) - (-32x^5-128x^3+128x^3+512x)}{(x^2+4)^4}$$

$$= \frac{16x^5+128x^3-768x}{(x^2+4)^4} = \frac{16x(x^4+8x^2-48)}{(x^2+4)^4} = \frac{16x(x^2+12)(x^2-4)}{(x^2+4)^4}$$



(e) Sketch the graph. Label the the local extrema and inflection points.

