# ON SIMPLE SPACES

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For a Banach space $X$ we denote by $\mathcal{L}(X)$ the algebra of all bounded linear operator on $X$. In this paper we are concerned with norm closed two sided ideals in $\mathcal{L}(X)$. Banach space $X$ is said to be ***simple*** if the set $\mathcal{K}(X)$ of all the compact operators is the only non-trivial proper closed ideal in $\mathcal{L}(X)$. It is known that the spaces $\ell_p$ $(1 \leqslant p < +\infty)$ and $c_0$ are simple (see, e.g., [CPY74]). No other simple spaces are currently known to the authors.

In this note We consider factorization ideals. In general, the set of all operators in $L(X)$ that factor through a given Banach space $Z$ need not be an ideal, as it might be not closed under addition. However, the set of all the operators in $\mathcal{L}(X)$ that factor through a power of $Z$ is an ideal. Denote this ideal by $\mathcal{J}_0^Z(X)$, and let $\mathcal{J}^Z(X)$ be the closure of $\mathcal{J}_0^Z(X)$ in $\mathcal{L}(X)$ in the operator norm. Thus, $\mathcal{J}^Z(X)$ is a closed ideal in $\mathcal{L}(X)$.

**Lemma 1.** *The following are equivalent*

(i) $\mathcal{J}_0^Z(X) = \mathcal{L}(X)$*;*
(ii) $I \in \mathcal{J}_0^Z(X)$*;*
(iii) $X$ *embeds complementably into a power of* $Z$*;*
(iv) $\mathcal{J}^Z(X) = \mathcal{L}(X)$

*Proof.* The implications (i)$\Leftrightarrow$(ii)$\Leftrightarrow$(iii)$\Rightarrow$(iv) are straightforward. The implication (iv)$\Rightarrow$(i) follows from the fact that the closure of a proper ideal in a unital Banach algebra is again a proper ideal (see, e.g., [Con90, Corollary VII.2.4]). $\square$

**Remark 2.** In the case when $Z \cong Z \oplus Z$, the ideal $\mathcal{J}_0^Z(X)$ coincides with the set of all the operators that factor through $Z$. Also, in this case the condition (iii) can be replaced with

(iii)′ $X$ embeds complementably into $Z$.

**Proposition 3.** *Suppose that $X$ is a Banach space and $V$ is a complemented subspace of $X$. If $X$ doesn't embed complementably into a power of $V$, then $X$ is not simple.*

*Proof.* $\mathcal{J}^V(X)$ is a closed ideal in $\mathcal{L}(X)$. It is proper by Lemma 1(iii). To show that $\mathcal{J}^V(X) \neq \mathcal{K}(X)$, let $P\colon X \to V$ be the canonical projection and let $S\colon V \to X$ be the inclusion map. Put $T = SP$, then $T \in \mathcal{J}^V(X)$, but $T_{|V} = \mathrm{id}_V$, so that $T$ is not strictly singular, hence $T \notin \mathcal{K}(X)$. □

**Corollary 4.** *If $X$ is simple then for every complemented subspace $V \subseteq X$, $X$ embeds complementably into a power of $V$.*

In view of Remark 2 we also get the following modification of Corollary 4, the proof is similar.

**Proposition 5.** *If $X$ is simple then $X$ embeds complementably into every its complemented subspace $V$ satisfying $V \cong V \oplus V$.*

**Corollary 6.** *Suppose that $X$ is simple and contains a complemented copy of $\ell_p$ ($1 \leq p < \infty$) or $c_0$. Then $X$ is isomorphic to $\ell_p$ or $c_0$.*

*Proof.* Indeed, if $V$ is a complemented subspace of $X$ which is isomorphic to $\ell_p$ or $c_0$, then, by Proposition 5, $X$ embeds complementably in $\ell_p$ or $c_0$. Then $X$ is isomorphis to $\ell_p$ or $c_0$. □

**Proposition 7.** *Suppose that there exists a non-strictly singular operator $T : X \to \ell_p$ for some $1 \leq p < \infty$. Then $X$ contains a complemented subspace isomorphic to $\ell_p$.*

*Proof.* Let $W$ be an infinite-dimensional subspace of $X$ such that $T_1 = T_{|W}$ is an isomorphism. Since every subspace of $\ell_p$ contains a complemented copy of $\ell_p$, then $T(W)$ contains a subspace $V$ isomorphic to *ell*$_p$. Let $P\colon \ell_p \to V$ be a bounded projection, then $T_1^{-1}PT$ is a bounded projection from $X$ to $T_1^{-1}V$. Hence, $T_1^{-1}V$ is a complemented subspace of $X$, isomorphic to $\ell_p$. □

**Corollary 8.** *Suppose that a simple Banach space $X$ have a subspace and a quotient isomorphic to $\ell_p$ ($1 \leq p < \infty$). Then $X$ is isomorphic to $\ell_p$.*

*Proof.* Let $q$ be the quotient map from $X$ onto $\ell_p$,and let $S$ be an isomorphism from $\ell_p$ onto a subspace $V$ of $X$. Put $T = Sq\colon X \to V$. We can view $T$ as an element of $L(X)$. Clearly, $T$ is not compact. Since $X$ is simple, then every strictly singular operator is copact, hence $T$ is not strictly singular either. Then it follows from Proposition 7 that $X$ contains a complemented copy of $\ell_p$, so that $X \cong \ell_p$ by Proposition 6. □

Next, we are going to apply previous results to Orlicz and Tsirelson spaces. Recall that an Orlicz function $M$ is a continuous non-decreasing and convex function defined for $t \geq 0$ such that $M(0) = 0$ and $\lim_{t\to\infty} M(t) = \infty$. Having an Orlicz function $M$, the Orlicz sequence space $\ell_M$ is the space of all scalars $x = (a_1, a_2, \ldots)$ such that $\sum_{n=1}^{\infty} M(|a_n|/\rho) < \infty$ for some $\rho > 0$, equipped with the norm

$$\|x\| = \inf\{\rho > 0 \mid \sum_{n=1}^{\infty} M(|a_n|/\rho) \leq 1\}.$$

We will be interested only in isomorphic properties of $\ell_M$, so we can assume, for simplicity, that $M(1) = 1$. In this case, the unit vectors $\{e_n\}_{n=1}^{\infty}$ form a normalized symmetric basic sequence in $\ell_M$.

By [LT77, Proposition 4.a.4], $\ell_M$ is separable $\iff$ $\ell_M$ does not contain $\ell_\infty$ $\iff$ $\{e_n\}_{n=1}^{\infty}$ is a boundedly complete basis for $\ell_M$.

Every Orlicz sequence space $\ell_M$ contains isomorphs of some $\ell_p$ or $c_0$. In fact, by [LT77, Theorem 4.a.9], the set of $p$'s for which $\ell_p \subset \ell_M$ is a closed interval (take $p = \infty$ for $c_0$). Moreover, [LT77, Theorem 4.b.3] Do we use these facts? asserts that *for a reflexive $\ell_M$, $\ell_p \subset \ell_M$ for some $p > 1$ if and only if $\ell_M$ has a quotient space isomorphic to $\ell_p$.*

**Corollary 9.** *Let $X = \ell_M$ be a simple Orlicz sequence space. Then $X$ is isomorphic to $\ell_p$ for some $1 \leq p < \infty$.*

Is there a simpler proof?

*Proof.* First we observe that $\ell_M$ must be separable. Indeed, otherwise $\ell_M$ contains $\ell_\infty$ (necessarily complemented because $\ell_\infty$ is injective). Since $\ell_\infty$ is not simple, $X$ cannot be simple either. (An argument for $\ell_\infty$ is not simple: $L_1(0,1)$ has a subspace isomorphic to $\ell_2$ and thus $\ell_2$ is a quotient of $L_\infty(0,1) \approx \ell_\infty$. Since $\ell_\infty$ also have subspace isomorphic to $\ell_2$ but not complemented, the result follows from Proposition 7.)

Let $\ell_M$ be a separable, simple Orlicz sequence space. Then $\{e_n\}_{n=1}^\infty$ is a symmetric boundedly complete basis for $\ell_M$. We know that there exists some $1 \leq p < \infty$ such that $\ell_p \subset \ell_M$ ($c_0 \not\subset \ell_M$ because the basis is boundedly complete).

If $p = 1$, then, since $\ell_1$ is injective in Banach spaces with an unconditional basis, we are done by Corollary 6.

Suppose $p > 1$. This implies that $\ell_M$ is reflexive (by James). Now by Lindenstaruss and Tzafriri result (mentioned above), $\ell_M$ has also a quotient isomorphic to $\ell_p$. Thus by Corollary 8, $X$ must be isomorphic to $\ell_p$. $\square$

Note that not every Orlicz sequence space contains a complemented copy of $\ell_p$. For the examples of such spaces, see [LT77, 4.c].

**Proposition 10.** *If $T$ is the Tsirelson space, then neither $T$ nor $T^*$ is simple. Indeed, let $V$ be the subspace generated by a lacunary enough subsequence of the basis.*

*Proof.* It follows from [CS89, Theorem VI.b.4] that there exists a fast increasing subsequence $n_k$ of integers such that the subspace $Z = \operatorname{span}\{e_{n_k}\}_k$ spanned by the corresponding subsequence of the basis does not contain a complemented subspace isomorphic to $T$. On the other hand, for every subspace $Z$ spanned by a subsequence we have $Z \cong Z \oplus Z$. Indeed, taking any subsequence $m_k$ such that $n_k < m_k < n_{k+1}$ we get another subspace, isomorphic to $Z$ and the subspace corresponding to a subsequence $(n_k, m_k)_k$ is also isomorphic to $Z$. Formally, letting

$Z_1 = \operatorname{span}\{e_{m_k}\}_k$ we have $Z_1 \cong Z$ and $Z \oplus Z_1 = \operatorname{span}\{e_{n_k}, e_{m_k}\}_k \cong Z$. Now Proposition 5 implies that $T$ is not simple.

Furthermore, notice that $Z^*$ is a complemented subspace of $T^*$, and $Z^* \cong Z^* \oplus Z^*$. Since $T$ is reflexive, we conclude that $T$ does not embed complementably into $Z^*$. Hence, $T^*$ is also not simple by Proposition 5.

□

If $Z$ is a Banach space, we use the following notation:

$$\ell_p(Z) = \Big(\bigoplus_{i=1}^{\infty} Z\Big)_p \qquad \text{and} \qquad \ell_p^n(Z) = \Big(\bigoplus_{i=1}^{n} Z\Big)_p.$$

Now consider $\mathcal{J}_0^{\ell_p(Z)}(X)$ and $\mathcal{J}^{\ell_p(Z)}(X)$. Since $\ell_p(Z) \cong \ell_p(Z) \oplus \ell_p(Z)$, Remark 2 implies that $\mathcal{J}_0^{\ell_p(Z)}(X)$ is the set of all operators that factor through $\ell_p(Z)$. Again, $\mathcal{J}^{\ell_p(Z)}(X)$ is proper if and only if $\mathcal{J}_0^{\ell_p(Z)}(X)$ is proper.

**Proposition 11.** *If $X$ is simple then $\ell_p(V) \cong \ell_p(X)$ for every complemented subspace $V$ in $X$.*

*Proof.* Suppose that $X$ is simple and $V$ is a complemented subspace of $X$. Then $\ell_p(V)$ is a complemented subspace of $\ell_p(X)$, so that $\ell_p(X) = \ell_p(V) \oplus Z_1$ for some $Z_1$.On the other hand, it follows from Corollary 4 that $X$ embeds complementably into $\ell_p(V)$, so that $\ell_p(X)$ embeds complementably into $\ell_p\big(\ell_p(V)\big) \cong \ell_p(V)$. Thus, $\ell_p(V) \cong \ell_p(X) \oplus Z_2$. Now, using Pełczyński's Decomposition method, one can prove a Schröder-Bernstein-type statement that $\ell_p(V) \cong \ell_p(X)$. Indeed,

$$\ell_p(X) \oplus \ell_p(V) \;\cong\; \ell_p(X) \oplus \ell_p(X) \oplus Z_2 \;\cong\; \ell_p(X) \oplus Z_2 \;\cong\; \ell_p(V),$$

and

$$\ell_p(V) \oplus \ell_p(X) \;=\; \ell_p(V) \oplus \ell_p(V) \oplus Z_1 \;\cong\; \ell_p(V) \oplus Z_1 \;=\; \ell_p(X).$$

□

Suppose that $T \in \mathcal{J}_0^Z(X)$, then $T$ factors through $Z^n$ for some $n$. Since $Z^n \cong \ell_p^n(Z)$, then $T$ factors through $\ell_p(Z)$. It follows that $\mathcal{J}_0^Z(X) \subseteq \mathcal{J}_0^{\ell_p(Z)}(X)$ and, therefore, $\mathcal{J}^Z(X) \subseteq \mathcal{J}^{\ell_p(Z)}(X)$.

**Proposition 12.** *Suppose that $X$ is simple, then $\mathcal{J}^Z(X)$ is proper if and only if $\mathcal{J}^{\ell_p(Z)}(X)$ is proper.*

How should we credit this theorem?

In the proof of Proposition 12 we will need the following result.

**Theorem 13.** *Suppose that $1 \leqslant p \leqslant \infty$ and $\{X_i\}$ is a sequence of infinite-dimensional Banach spaces. Put $X = \left(\bigoplus_{i=1}^{\infty} X_i\right)_p$ if $p < \infty$, or $X = \left(\bigoplus_{i=1}^{\infty} X_i\right)_{c_0}$ if $p = \infty$. If $Y$ is a complemented subspace of $X$ then either $\ell_p$ (respectively, $c_0$) embeds complementably into $Y$, or $Y$ actually embeds complementably into a finite sum $\left(\bigoplus_{i=1}^{n} X_i\right)_p$ for some $n$.*

*Proof of Proposition 12.* Since $\mathcal{J}^Z(X) \subseteq \mathcal{J}^{\ell_p(Z)}(X)$ is always true, we only need to prove that if $\mathcal{J}^{\ell_p(Z)}(X) = \mathcal{L}(X)$ then $\mathcal{J}^Z(X) = \mathcal{L}(X)$. In view of Lemma 1 it suffices to show that if $\mathcal{J}_0^{\ell_p(Z)}(X) = \mathcal{L}(X)$ then $\mathcal{J}_0^Z(X) = \mathcal{L}(X)$. Suppose $\mathcal{J}_0^{\ell_p(Z)}(X) = \mathcal{L}(X)$, then by Lemma 1 $X$ embeds complementably into $\ell_p(Z)$. By Theorem 13 and Corollary 6 we conclude that $X$ embeds complementably into $\ell_p^n(Z)$ for some $n$, so that $\mathcal{J}_0^Z(X) = \mathcal{L}(X)$ by Lemma 1. $\square$

*Proof of Theorem 13.* For an integer $n \geqslant 1$, let $P_n : X \to X$ be the natural projection onto $\left(\bigoplus_{i=1}^{n} X_i\right)_p$. By $Q\colon X \to X$ denote a bounded projection onto the subspace $Y$. The proof splits into two cases.

**(I):** $\exists\delta\ \exists m_0\ \|QP_{m_0}y\| \geqslant \delta\|y\|$ for all $y \in Y$.

**(II):** $\forall\varepsilon > 0\ \forall m\ \exists y \in Y\ \|QP_m y\| \leqslant \varepsilon\|y\|$.

**Case (I):** First note that

$$\|P_{m_0}y\| \geqslant \delta\|y\| \qquad \text{for all } y \in Y. \tag{1}$$

Set $F = P_{m_0}(Y)$ and $T = P_{m_0}|Y$. Then (1) immediately implies that $T\colon Y \to F$ is an isomorphism from $Y$ onto $F$ and $\|T^{-1}\| \leqslant \delta^{-1}$.

Let $S = P_{m_0}Q|F$. Then $S\colon F \to F$ and we have

$$\|Sf\| \geqslant \delta^2\|f\| \qquad \text{for all } f \in F. \tag{2}$$

Indeed, let $f \in F$ and let $f = P_{m_0}y$ for some $y \in Y$. Then, by (1),

$$\|Sf\| \geqslant \delta\|Qf\| = \delta\|QP_{m_0}y\| \geqslant \delta^2\|y\| \geqslant \delta^2\|f\|.$$

Thus (2) implies $\|S^{-1}\| \geqslant \delta^{-2}$. Consider the operator $\widetilde{Q} = S^{-1}P_{m_0}Q\colon X \to F$. Clearly, $\widetilde{Q}$ is a projection from $X$ onto $F$ and $\|\widetilde{Q}\| \leqslant \delta^{-2}\|Q\|$. Thus $F$ is isomorphic to $Y$ and complemented in $X$, in this case.

**Case (II):** Let $R_n = Id - P_n$. First observe that if a vector $y \in Y$ satisfies condition (II) then we also have

$$\|R_m y\| \geqslant (1-\varepsilon)\|Q\|^{-1}\|y\| =: c\|y\|. \tag{3}$$

Indeed, $\|QR_m y\| \geqslant \|Qy\| - \|QP_m y\| \geqslant (1-\varepsilon)\|y\|$; and on the other hand, $\|QR_m y\| \leqslant \|Q\|\,\|R_m y\|$.

Assume for simplicity that $\varepsilon = 0$ in condition (II) and that all vectors and functionals involved in the remaining part of the argument have always finite support. The general case follows from this by a standard approximation. Under this additional assumption, condition (II) says

**(II'):** $\forall m\ \exists y \in Y\ QP_m y = 0.$

Construct by induction a sequence of integers $0 = n_0 < n_1 < n_2 < \dots$ and sequences of vectors $\{u_j\}$ in $Y$ and of functionals $\{u_j^*\}$ in $X^*$ such that for every $j = 1, 2, \dots$ we have

**(i):** $\|u_j\| = 1$, and $QP_{n_{j-1}}u_j = 0$, $R_{n_j}u_j = 0$, and $\|R_{n_{j-1}}u_j\| \geqslant c$;
**(ii):** $\|u_j^*\| = 1$, $u_j^*(u_j) \geqslant c$, and $P_{n_{j-1}}u_j^* = 0$, $R_{n_j}u_j^* = 0$, and $R_{n_j}Q^*u_j^* = 0$.

Indeed, given $n_{j-1}$, let $u_j$ be any norm 1 vector satisfying condition (II') for $m = n_{j-1}$, and let $n'$ be such that $R_{n'}u_j = 0$. Then by (3), conditions (i) are satisfied whenever $n_j \geqslant n'$. Let $u_j^*$ be a norming functional for $R_{n_{j-1}}u_j$, so that $P_{n_{j-1}}u_j^* = 0$, $R_{n'}u_j^* = 0$ and $\|u_j^*\| = 1$, and $u_j^*(u_j) = u_j^*(R_{n_{j-1}}u_j) = \|R_{n_{j-1}}u_j\| \geqslant c$. Finally, let $n_j \geqslant n'$ be an integer such that $R_{n_j}Q^*u_j^* = 0$. It is clear that (ii) is satisfied.

We shall show that $\{u_j\}$ is equivalent to the unit vector basis in $\ell_p$, and its span is complemented in $X$.

Set $w_j = R_{n_{j-1}}u_j$, for $j = 1, 2, \dots$. By (i) we have $Qu_j = Qw_j$ and $w_j$ is supported on the (open) interval $(n_{j-1}, n_j)$, for $j = 1, 2, \dots$. Let

$\{a_j\}$ be a finite sequence of scalars. Then

$$\begin{aligned}(4)\qquad \Big\|\sum_j a_j u_j\Big\| &= \Big\|\sum_j a_j Q u_j\Big\| = \|\sum_j a_j Q w_j\| \\ &\leqslant \|Q\|\Big\|\sum_j a_j w_j\Big\| \leqslant \|Q\|\Big(\sum_j |a_j|^p\Big)^{1/p}.\end{aligned}$$

To get the lower $\ell_p$-estimate, fix a finite sequence of scalars $\{a_j\}$. For an arbitrary finite scalar sequence $\{b_j\}$ set $x^* = \sum_j b_j u_j^*$, and consider $x^*(\sum_k a_k u_k)$.

Fix $k = 1, 2, \ldots$. Since $R_{n_k} u_k$ and $u_j^* = R_{n_k} u_j^* = 0$ for $j > k$, we infer that $u_j^*(u_k) = 0$ for all $j > k$. For $j < k$ we have, by conditions (i) and (ii),

$$\begin{aligned} u_j^*(u_k) &= u_j^*(Q u_k) = Q^* u_j^*(u_k) \\ &= P_{n_j} Q^* u_j^*(u_k) = u_j^*(Q P_{n_j} u_k) = 0. \end{aligned}$$

Thus, $x^*\big(\sum_k a_k u_k\big) = \sum_k a_k b_k u_k^*(u_k)$. Since $u_k^*(u_k) \geqslant c$ for all $k$, choosing an appropriate sequence $\{b_k\}$ with $\sum_k |b_k|^{p'} = 1$ we get

$$c\Big(\sum_k |a_k|^p\Big)^{1/p} \leqslant \sum_k a_k b_k u_k^*(u_k) \leqslant \|x^*\|\Big\|\sum_k a_k u_k\Big\|.$$

(Here $1/p + 1/p' = 1$, for $p > 1$ and $p' = \infty$, for $p = 1$, with an appropriate interpretation of the $\ell_{p'}$-norm.)

Finally note that since $u_j^*$ is supported on the (open) interval $(n_{j-1}, n_j)$, for $j = 1, 2, \ldots$, for any finite scalar sequence $\{b_j\}$ we have $\|\sum_j b_j u_j^*\| \leqslant \Big(\sum_j |b_j|^{p'}\Big)^{1/p'}$; hence $\|x^*\| \leqslant 1$. Thus

$$c\Big(\sum_k |a_k|^p\Big)^{1/p} \leqslant \Big\|\sum_k a_k u_k\Big\|.$$

To show that $\operatorname{span}\{u_j\}$ is complemented, set $w_j^* = u_j^*/u_j^*(w_j)$, for $j = 1, 2, \ldots$. Then $\|w_j^*\| \leqslant 1/c$ for all $j$, and for any finite scalar sequence $\{b_j\}$ we have

$$(5)\qquad \Big\|\sum_j b_j w_j^*\Big\| \leqslant (1/c)\Big(\sum_j |b_j|^p\Big)^{1/p}.$$

Define an operator $\widetilde{Q} : X \to \operatorname{span}\{u_j\}$ by $\widetilde{Q}x = \sum w_j^*(x)u_j$, for $x \in X$. For all $j$ we have $u_j^* = R_{n_{j-1}}u_j^*$, and hence $w_j^*(u_j) = 1$. As before, $u_j^*(u_k) = 0$ whenever $k \neq j$, $j, k = 1, 2, \ldots$. Thus $\widetilde{Q}$ is a projection. The following estimate for the norm $\|\widetilde{Q}\|$ is standard.

Let $x \in X$. For an appropriate scalar sequence $\{b_j\}$ satisfying $\sum_j |b_j|^p = 1$ we have, using inequalitiees (4) and (5),

$$\begin{aligned} \|\widetilde{Q}x\| &\leqslant \|Q\| \Big(\sum_j |w_j^*(x)|^{p'}\Big)^{1/p'} = \|Q\| \Big|\sum b_j w_j^*(x)\Big| \\ &\leqslant \|Q\| \Big\|\sum_j b_j w_j^*\Big\| \, \|x\| \leqslant (1/c)\|Q\| \, \|x\|. \end{aligned}$$

So $\|\widetilde{Q}\| \leqslant (1/c)\|Q\|$, and this completes the proof. □

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