Spirographs

Purpose: The purpose of this project is to reinforce your understanding of parametric equations.

Procedure: Read the outline below and derive the required formulas. After completing the derivation test your answer by using Mathematica (or some other graphing program or calculator) to plot the graphs. Then write your results **neatly**. Be sure to include relevant graphs in what you turn in. Your grade will be determined in part by your presentation. Include a careful explanation of how you derived the formulas with relevant pictures.

Project Outline: Katie's little brother Kelly has a spirograph which he uses to make pretty designs. There are two problems with the spirograph. After Kelly gets everything all lined up and starts to draw the design, either the gear slips or else the pen quits in the middle of the design. One day while Katie was working on the computer Kelly had almost completed a beautiful picture when the gear slipped. Kelly started to cry. Katie promised Kelly that if he would tell Katie which gear he was using then Katie could write a computer program to print out the design. Katie promised to have a program written within two weeks that would print out any of the designs Kelly would want which involve two circles. That night after Kelly went to bed Katie realized that she needed some sort of an equation so she could graph it.

The next day Kelly asked Katie if she knew how long the curve traced by the pen is when the moving circle goes around the fixed circle once. Not wanting to look stupid to her little brother, Kelly said that she could compute the length if Kelly told her the radii of both circles and where he placed the pen, but she didn't have time right now to do it. After she finishes the program she will work it out. Kelly said he wanted to know the distance when both radii were 1, the outer circle moves and the pen is placed on the edge of the circle. He then asked if placing the pen in the center of the circle always gives the shortest path when the outer moving circle goes around exactly once, regardless of the radii of the circles.

After thinking about the problem Katie started to panic. She then remembered that you were taking calculus so she called and asked for help. It is your job to find a parametric equation whose graph gives the desired design. Then, based on the equations, compute the desired path length and either verify Kelly's guess that the center always gives the minimum path length or else show that it doesn't always.

In case you are not familiar with a spirograph, the designs are created by rolling one circle either inside or outside a fixed circle with the pen attached to the rolling circle. See Figure 1. Note that the pen's positon can be fixed anywhere inside the rotating circle.

You should show the derivation of the equation for both cases. To check your answer, plot your equations using Mathematica, a graphing calculator, or another computer software package. When determining arc length it will be possible to calculate the integral exactly, at least in the case where Kelly wants the answer. You may wish to do some experimentation using numerical integration to see if Kelly's guess is correct.

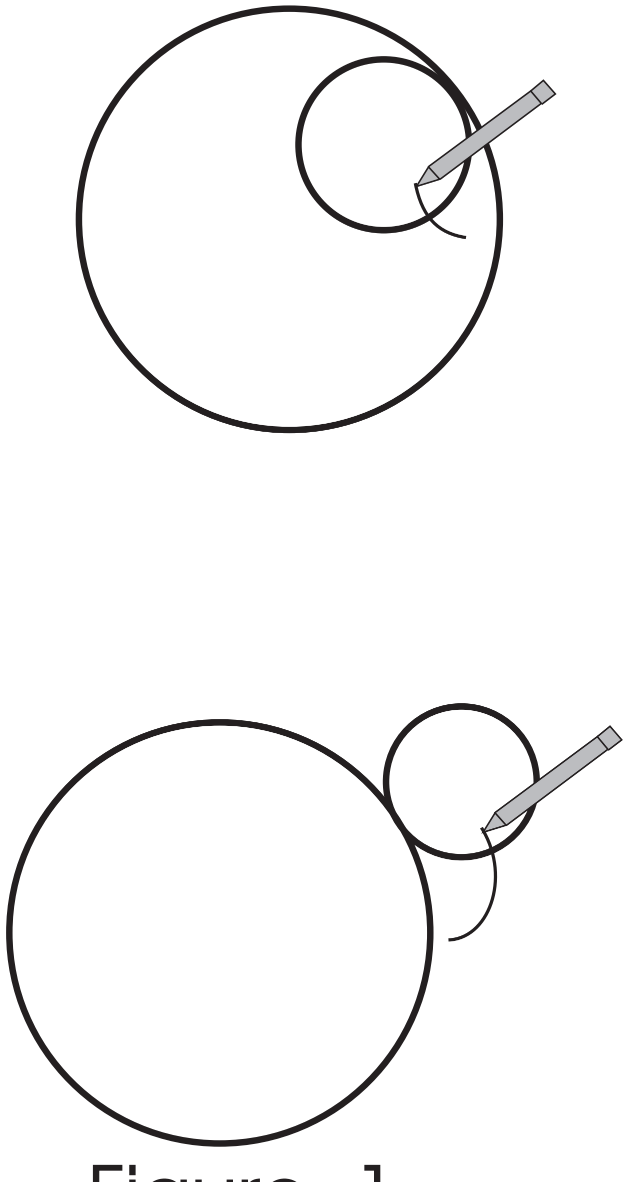

Figure 1.