Tibetan Spindles

Purpose. The first purpose of this project is to apply integration to a real world situation in order to design a useful product. The second purpose is to investigate how Riemann sums and their limits can be used to compute quantities that are useful in physics.

The story. Fleegle is an experienced spinner and she wishes to add a new and interesting spindle to her collection. Fleegle likes Tibetan spindles. Tibetan spindles are characterized as having a shaft to wind the yarn on and a whorl to keep the spindle spinning. She wants her new spindle to have similar spin characteristics to her favorite Tibetan spindles, but she would like the shape of the whorl to be different and she wants an atrtractive wood combination. Knowing that you are both an expert in calculus and very artistic, she asks for your help. Fleegle gives you the dimensions and the type of wood used for a couple of spindles that spin well and she wants the spindle that you design to spin as well as these. Your job is to design a spindle for Fleegle.

Procedure. You are to follow the outline below.

1. The moment of inertia I is the main characteristic that determines how an object spins. For a point mass of m that is a distance of r from the axis of rotation, the moment of inertia is mr^2. For any finite number of point masses, the total moment of inertia is the sum of each. For a solid object, such as a top or spindle, one needs to take limits of Riemann sums to work out the integral to find the moment of inertia. For this part, find the formula for the moment of inertia of a solid with density ρ formed by rotating the region in the plane bounded by the $x-$axis, $y = f(x)$, $x = a$ and $x = b$ about the $y-$axis. (Assume only that $f(x) \geq 0$ and f is continuous at x for all $a \leq x \leq b$.)
2. Find the mass and moment of inertia of a cylinder with radius r, height h and density ρ whose axis of rotation is the axis of the cylinder. Show that for a cylinder, $I = \frac{Mr^2}{2}$ where M is the mass of the cylinder.
3. Find the moment of inertia of a cone with radius r, height h and density ρ. Find a formula for the moment of inertia of a cone that involves only the mass and the radius of the cone (similar to the formula in part 2).
4. Next find the mass and the moment of inertia of a solid with density ρ whose shape is the solid of revolution obtained by rotating the area in the first quadrant bounded by the parabola $y = -ax^2 + h$ and the coordinate axes.
5. Use your results in part 4) to show that for a parabolic shaped whorl, $I = \frac{Mr^2}{3}$ where M is the mass and r is the radius of the whorl.
6. Next find the mass and the moment of inertia of a spherical shaped whorl having radius r and density ρ. (These spindles are called bead spindles.) Then use your formulas to get a formula for I that involves only the mass and the radius (not the density) of the whorl.
7. Fleegle tells you that she has a spindle whose spin she really likes and it has a parabolic whorl made of snakewood with height 2.0 cm and radius 2.2 cm. She also has a nice bead spindle whose bead has radius 2 cm and mass 17 g. Find the moment of inertia

of each whorl. Based on the fact that these spindles spin well, approximately what moment of inertia do you think you should try for when designing a good spindle?

8. The shaft of a typical spindle is roughly a cone with radius no more than 0.5 cm, height no more than 25 cm and density no more than 1.4 g/cc. Based on this, give an estimate of the moment of inertia for a typical shaft. How do the moments of inertia of a shaft and the whorls in part 7 compare?
9. Suppose that you wanted to make a spindle with an ebony parabolic whorl. If the height of the whorl is to be 2.0 cm, what should the diameter be? Next suppose you wanted to make a spherical whorl out of holly, what radius would you use?
10. Now it is time to design your own spindle. You are to choose attractive woods for the whorl and the shaft as well as a nice shape for the whorl, keeping in mind that the whorl must have a good moment of inertia. The shape should be a solid of revolution. The function rotated may be defined piecewise, as one formula, or simply a drawing with distances marked. However you specify the function, you will need to show the computations for the moment of inertia. You should at least specify the wood to be used for the shaft and if you wish, you may specify the shaft's shape as well. The spindle should be between 22 and 28 cm long.

Helpful web site:

http://www.hobbithouseinc.com/personal/woodpics

http://www.engineeringtoolbox.com/wood-density-d_40.html

http://www.ravelry.com Look at the forums Spindle Candy, Support Spindlers, and Spindle Lore

Note: I plan to make your spindle over the break. I should have no trouble finding any of the following woods:

Walnut, Maple, Ebony, Cherry, Zebrawood, Tulipwood, Canarywood, Mesquite, Bois d'arc, Sycamore, Bocote, Macacauba, Teak, Yellowheart, Purpleheart, Redheart, Bloodwood, Holly, Ziricote, Limba (white or black), Kingwood, Oak burl (whorl only, not shaft), Jatoba (shaft only, not whorl), Ironwood (shaft only, not whorl), Mahogany, African Blackwood, and Goncalo Alves.

If you specify a different wood, I will try to get it, but I may have to make a substitution.

I plan to make one spindle for each group design. Your group may either keep the spindle or donate it to raise money for a UNT math scholarship. If your group decides to keep the spindle, then I may make a second one to sell to raise scholarship money.